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ON I-FINITE LEFT QUASI-DUO RINGS

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Abstract. A ring is called left quasi-duo (left QD) if every maximal left ideal is a right ideal, and it is called I-finite if it contains no infinite orthogonal set of idempotents. It is shown that a ring is I-finite and left QD if and only if it is a generalized upper-triangular matrix ring with all diagonal rings being division rings except the lower one, which is either a division ring or it is I-finite, left QD and left 'soclin' (left QDS). Here a ring is called left soclin if each simple left ideal is nilpotent. The left QDS rings are shown to be finite direct products of indecomposable left QDS rings, in each of which $1 = f_1 + \cdots + f_m$ where the f_i are orthogonal primitive idempotents, with $f_k \approx f_l$ for all k, l, and $\approx$ is the block equivalence on $\{f_1, \ldots, f_m\}$.

A ring is shown to be left soclin if and only if every maximal left ideal is left essential, if and only if the left socle is contained in the left singular ideal. These left soclin rings are proved to be a Morita invariant class; and if a ring is semilocal and non-semisimple, then it is left soclin if and only if the Jacobson radical is essential as a left ideal.

Left quasi-duo elements are defined for any ring and shown to constitute a subring containing the centre and the Jacobson radical of the ring. The 'width' of any left QD ring is defined and applied to characterize the semilocal left QD rings, and to clarify the semiperfect case.

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